

Possible daily and seasonal variations in quantum interference induced by Chern-Simons gravity

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Abstract

Possible effects of Chern-Simons (CS) gravity on a quantum interferometer turn out to be dependent on the latitude and direction of the interferometer on the Earth in orbital motion around the Sun. Daily and seasonal variations in phase shifts are predicted with an estimate of the size of the effects, wherefore neutron interferometry with ~ 5 meters arm length and $\sim 10^{-4}$ phase measurement accuracy would place a bound on a CS parameter comparable to Gravity Probe B satellite.

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Introduction.— It has long been a fundamental issue to understand the interplay between the quantum theory and the gravitational physics. The interplay is studied mostly by theoretical experiments [1]. Corella, Overhauser, and Werner (COW) [2] succeeded an first experiment involving both the Plank constant h and the gravitational constant G by using a neutron interferometer. In COW experiments, a neutron interferometer is tilted, such that a neutron beam path I is higher above the surface of the Earth than the other path segment II, causing a gravitationally induced phase shift of the neutron de Broglie waves on path II relative to path I. The gravitationally induced phase shift was experimentally observed [2, 3]. In recent years, technological progress has been brought into quantum experiments including neutron interferometers and quantum optics. Current attempts to probe general relativistic effects in quantum mechanics focus on precision measurements of phase shifts in quantum interferometers (e.g. [4]). Hogan has recently proposed an ambitious idea to use quantum interferometers as an experimental probe of a quantum spacetime at the Planck scale [5]. Quantum experiments may play a role in probing an intermediate regime between general relativistic gravity and Planck scale physics.

Current astronomical observations, such as the apparent accelerated expansion of the Universe, suggest a possible infrared modification to general relativity (GR). The Chern-Simons (CS) correction is not an *ad hoc* extension, but it is actually motivated by both string theory, as a necessary anomaly-canceling term to conserve unitarity [6], and loop quantum gravity [7]. Alexander and Yunes have recently pointed out that CS gravity possesses the same parameterized post-Newton (PPN) parameters as general relativity, except for the inclusion of a new term, proportional to the CS coupling and the curl of the PPN vector potential [8, 9]. They have also shown that this new correction might be used in *classical* experiments, such as Gravity Probe B, to bound CS gravity and test string theory (See [10] for an extensive review of CS modified gravity).

In contrast to approaches focusing on general relativistic effects on quantum systems [1], we shall study CS gravity in quantum experiments as another attempt to probe quantum gravity. Nandi and his collaborators [11] have recently discussed the quantum phase shift in Chern-Simons modified gravity, where an isolated gravitating body was considered. They have concluded that the induced shifts by the spin of the body are too tiny to be observed. However, the Earth's orbital angular momentum ($\sim 3 \times 10^{40} \text{ kg} \cdot \text{m}^2\text{s}^{-1}$) is much larger than its spin angular momentum ($\sim 7 \times 10^{33} \text{ kg} \cdot \text{m}^2\text{s}^{-1}$). Both of the axial vectors may play

a role in CS gravity. Therefore, we consider gravitationally interacting bodies in order to investigate the quantum mechanical effects of the Earth's orbital angular momentum in CS gravity. The main result of this Letter suggests that a CS modified gravity theory may predict daily and seasonal phase shifts in quantum interferometers, which are in principle distinct from the general relativistic effects. This feature can be currently used as a *quantum* tool to probe CS gravity.

CS gravity.— CS gravity modifies GR via the addition of a correction to the Einstein-Hilbert action, namely [12, 13]

$$S_{CS} = \frac{1}{16\pi G} \int d^4x \frac{1}{4} f R^* R, \quad (1)$$

where f is a prescribed external field (with units of area in geometrized units) that acts as a coupling constant, R is the Ricci scalar, and the star stands for the dual operation.

The weak-field solution to the CS modified field equations in PPN gauge is given by [8–10]

$$g_{00} = -1 + 2U - 2U^2 + 4\Phi_1 + 4\Phi_2 + 2\Phi_3 + 6\Phi_4 + O(6), \quad (2)$$

$$g_{0i} = -\frac{7}{2}V_i - \frac{1}{2}W_i + 2\dot{f}(\nabla \times V)_i + O(5), \quad (3)$$

$$g_{ij} = (1 + 2U)\delta_{ij} + O(4), \quad (4)$$

where $U, \Phi_1, \Phi_2, \Phi_3, \Phi_4, V_i, W_i$ are PPN potentials (e.g. [14]), $O(A)$ stands for PN remainders of order $O(1/c^A)$ for the light speed c and the dot denotes the derivative with respect to $x^0 \equiv ct$. Note that this is a non-dynamical (kinematical) model of CS modified gravity, where we assume that f depends on time only. The non-dynamical CS theory is tractable and could become a good approximation in weak fields, regardless of a possible evolution problem of the external field f (presumably near the central region) consistent with Pontryagin constraint. A full dynamical study of seeking approximate solutions for rotating extended bodies has yet to be carried out [10]. Henceforth, we investigate whether \dot{f} term in Eq. (3) brings new gravitational physics into quantum systems.

Following [8], let us consider a system of nearly spherical bodies in the standard PPN point-particle approximation. A follow-up study conducted by Smith and his collaborators [15] shows that the new term in Eq. (3) is valid even outside of a weakly gravitating spinning body like the Earth. For the above vector potential V_i , the CS correction to the

metric becomes in the barycenter frame [8, 9, 16, 17]

$$\delta_{CS}g_{0i} = \frac{2G}{c^3} \sum_A \frac{\dot{f}}{r_A} \left[\frac{m_A}{r_A} (\vec{v}_A \times \vec{n}_A)^i - \frac{J_A^i}{2r_A^2} + \frac{3}{2} \frac{(\vec{J}_A \cdot \vec{n}_A)}{r_A^2} n_A^i \right], \quad (5)$$

with m_A the mass of the Ath body, r_A the field point distance to the Ath body, $n_A^i = x_A^i/r_A$ a unit vector pointing to the Ath body, v_A^i the velocity of the Ath body, J_A^i the spin-angular momentum of the Ath body, and the \cdot and \times operators are the flat-space inner and outer products. Note that the CS correction couples with the spin and the orbital angular momenta.

Phase shifts.— We consider a quantum interferometer that consists of a closed path C (its area S) on the Earth, as shown by Fig 1.

The Hamiltonian for a quantum particle in a curved spacetime involves $g_{\mu\nu}$. The linear-order correction to the Hamiltonian by g_{0i} becomes $\delta H = mcg_{0i}v^i$ for a slowly-moving particle [18]. A phase difference induced by g_{0i} is thus expressed as [3]

$$\begin{aligned} \Delta &= \frac{1}{\hbar} \oint_C \delta H dt \\ &= \frac{mc}{\hbar} \oint_C \vec{g} \cdot d\vec{r}, \end{aligned} \quad (6)$$

where \vec{g} denotes (g_{01}, g_{02}, g_{03}) , m denotes the quantum particle mass, $\hbar \equiv h/2\pi$ denotes Dirac's constant. By using Stokes theorem, Δ is rewritten in the surface integral form over S as

$$\Delta = \frac{mc}{\hbar} \int_S (\vec{\nabla} \times \vec{g}) \cdot d\vec{S}. \quad (7)$$

This form has an analogy in the Aharonov-Bohm (AB) effect. The AB effect in the phase shift, which was confirmed experimentally [19], is $\propto \oint_C \vec{A} \cdot d\vec{r} = \int_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{S}$ for a vector potential \vec{A} in the electromagnetism. Note that the phase difference Δ in Eq. (7) is caused by time dilation and hence it does not depend on de Broglie wavelength λ , in contrast to COW experiments.

Let us substitute the CS term of Eq. (5) into Eq. (7) to obtain Δ for CS gravity. By using an identity $\epsilon^{ijk}(1/r)_{,jkl} = 0$ with the Levi-Civita symbol ϵ^{ijk} , one can see that the J -dependent part of the metric in Eq. (5) always vanishes in Eq. (7), whereas the v -dependent part makes contributions.

Since Δ involves the curl operation on the surface of the Earth and the Earth radius r_E is much shorter than 1AU, the terms associated with the solar mass M_\odot in Eq. (7) is

$O(M_\odot M_E^{-1} r_E^3 1\text{AU}^{-3}) \sim 10^{-9}$ smaller than those with the Earth's mass M_E , so that the terms with the solar mass (and other planetary ones) can be safely neglected. Henceforth, we focus on the Earth mass (also its spin and orbital angular momentum) in CS gravity. Hence, Eq. (7) becomes

$$\begin{aligned}\Delta_{CS} &= \frac{2m}{\hbar c^2} \int_S \dot{f} \frac{GM_E}{r^3} [3(\vec{v}_E \cdot \vec{n}_E) \vec{n}_E - \vec{v}_E] \cdot \vec{N}_I dS \\ &= 2\dot{f} \frac{mGM_E S}{\hbar c^2 r_E^3} [3(\vec{v}_E \cdot \vec{n}_E) \vec{n}_E - \vec{v}_E] \cdot \vec{N}_I,\end{aligned}\quad (8)$$

where we used $r_E \gg \sqrt{S}$ (the Earth radius is much larger than the interferometer arm length) and hence $r = r_E$ in the integrand. Here, in an inertial frame, \vec{v}_E denotes the Earth's orbital velocity, \vec{n}_E stands for the unit vertical vector on the ground (at a certain latitude), \vec{N}_I means the unit normal to the interferometer plane (See also Fig. 1). The unit normal vectors \vec{n}_E and \vec{N}_I in an inertial frame change with time as the Earth rotates. The change rate depends on the latitude. Moreover, \vec{N}_I depends also on the interferometer's direction such as horizontal and vertical. In contrast to COW experiments, the interferometer direction such as North and East does matter in CS gravity. Therefore, the factor $[3(\vec{v}_E \cdot \vec{n}_E) \vec{n}_E - \vec{v}_E] \cdot \vec{N}_I$ in Eq. (8), depending on the latitude and direction, changes with the Earth's spin and orbital motion.

In order to see more explicitly the interplay between quantum mechanics and CS gravity, the magnitude of Eq. (8) is factored as

$$|\Delta_{CS}| \sim 4 \left(\frac{mc^2}{\hbar} \right) \left(\frac{\dot{f}}{c} \frac{GM_E}{c^2 r_E} \frac{v_E}{c} \right) \left(\frac{S}{r_E^2} \right), \quad (9)$$

where $[3(\vec{v}_E \cdot \vec{n}_E) \vec{n}_E - \vec{v}_E] \cdot \vec{N}_I \sim 2v_E$. It is worthwhile to mention that the first fraction in the right hand side of Eq. (9) is due to the quantum mechanical physics and it is large enough $\sim 10^{24} \text{s}^{-1}$ to compensate the factor in the second parenthesis due to the CS gravitational effect $\sim \dot{f} c^{-1} \times 10^{-14}$, where m is neutron mass. The last factor in Eq. (9) is the squared ratio of the interferometer arm length (often $\sim 60 \text{cm}$) to the Earth radius. In total, the magnitude of Δ_{CS} is

$$|\Delta_{CS}| \sim 10^{-3} \text{s}^{-1} \times \left(\frac{mc^2}{1\text{GeV}} \right) \left(\frac{\dot{f}}{c} \right) \left(\frac{S}{0.4\text{m}^2} \right). \quad (10)$$

On the other hand, COW experiments that measure phase shifts due to Newton gravity rely on the inclination angle of the interferometer but not on the latitude [2]. Moreover, the general relativistic effects of a slowly rotating object, known as the Lense-Thirring effects,

cause a phase shift proportional to $\vec{\omega}_E \cdot \vec{S}$ (e.g. [20, 21]), wherefore they depend only on the angle between the Earth's axis and the interferometer. Hence, the directional dependence of the general relativistic phase shifts is in principle distinct from that of CS gravity.

Figures 2 and 3 show numerical calculations of time variations in the phase difference for the Earth's parameters such as the inclination angle of the Earth's axis I_E , the mean orbital angular velocity Ω_E , the spin rate ω_E , where the eccentricity of the Earth orbit makes a tiny input. The magnitude of the variations in the phase shift by the inclination of the Earth's axis is expected to be $\sim O(\sin I_E) \sim O(0.1)$.

Finally, we mention whether other quantum gravity effects could be present in the phase shift. Eq. (1) is the first Parity-violating term in a series of curvature corrections. There are probably other terms that would be cubic and higher order corrections. The next term would induce a correction $\sim \dot{f}^2(\nabla \times V)^2$ in Eq. (3), so that the part in the second parenthesis of Eq. (9) could include $\dot{f}^2 c^{-2} 10^{-30}$ at the next order. Hence, the next (and higher order) terms can be safely neglected. Other couplings motivated by quantum gravity on the quantum interference are left as a future work.

Discussion and Conclusion.— We considered effects of CS gravity on a quantum interferometer. The CS effect in the phase shift has an analogy in the AB one. The CS effects turn out to be dependent on the latitude and direction of the interferometer on the Earth in orbital motion around the Sun. The CS dependence is different from the general relativistic one. Daily and seasonal variations in phase shifts, independent of the wavelength, are thus suggested with an estimate of the size of the effects. Numerical studies for an interferometer at middle latitudes are left as a future work [22].

Current measurements of phase shifts in neutron interferometry do not report any anomalous (daily nor seasonal) variations with phase measurement accuracy at $O(10^{-3})$. Current neutron interferometry, therefore, places a bound on CS gravity as $\dot{f}c^{-1} < 10^0\text{s}$ ($\dot{f} < 10^5\text{km}$), which is worse by three digits than the constraint $\dot{f}c^{-1} < 10^{-3}\text{s}$ by the classical experiment GPB (Gravity Probe B) [8, 23] and also LAGEOS [15]. Future progress in quantum technology may improve the bound. A bound comparable to the GPB limit would be placed, if neutron interferometry were sufficiently improved for $\Delta \times S^{-1}$ (nearly by three digits), for instance ~ 5 meters arm length and $\sim 10^{-4}$ phase measurement accuracy. It is awaited. Experimental setups usually suffer from many other seasonal variations. Lacking a signal, a constraint may be placed on \dot{f} . In the presence of a signal, on the other hand, one would

have to eliminate all other possible sources of seasonal variability.

Finally, we mention briefly a possible path toward the desired technology improvement. Neutron interferometers are typically made from a single large crystal of silicon, 20 to 60 cm or more in length. Modern semiconductor technology allows large single-crystal silicon boules to be easily grown [24]. In the near future, therefore, neutron interferometry with a few-meter arm length might become available. For improving the phase measurement accuracy, technological challenges are in progress. For observations of the topological Aharonov-Casher effect, Werner and his collaborators have already obtained the result for the measured phase shift with a one-sigma statistical error bar as $\pm 0.34 \text{ mrad} \sim O(10^{-4})$, where approximately, a total of 500 000 000 neutrons were counted in the interferograms over a period of 2 years (See [25] for a review of observations of Aharonov-Bohm effects by neutron interferometry). Hence, CS gravity effects depending on the direction (such as the northeast) of the interferometer could be tested, if the Werner's interferometer had a longer arm (a few meters) and rotated. Next, the CS dependence on the latitude might be tested, if another interferometer with the same capability were built in a different latitude (for instance on the equator).

Furthermore, Seki and his collaborators have recently developed a multilayer cold-neutron interferometer and experimentally the phase measurement accuracy of 0.01 rad, where they used only 1.5×10^5 neutrons in a short time ~ 49 hours [26]. Such a technology might be used for a future experimental test of CS-type seasonal variations. However, it seems insufficient for experimental tests of daily variations. For a daily variation test, a breakthrough in neutron interferometry is needed. Motivated by quantum computations, for instance, Pushin and his collaborators have demonstrated experimentally how quantum-error-correcting codes may be used to improve experimental designs of quantum devices to achieve noise suppression in neutron interferometry [27].

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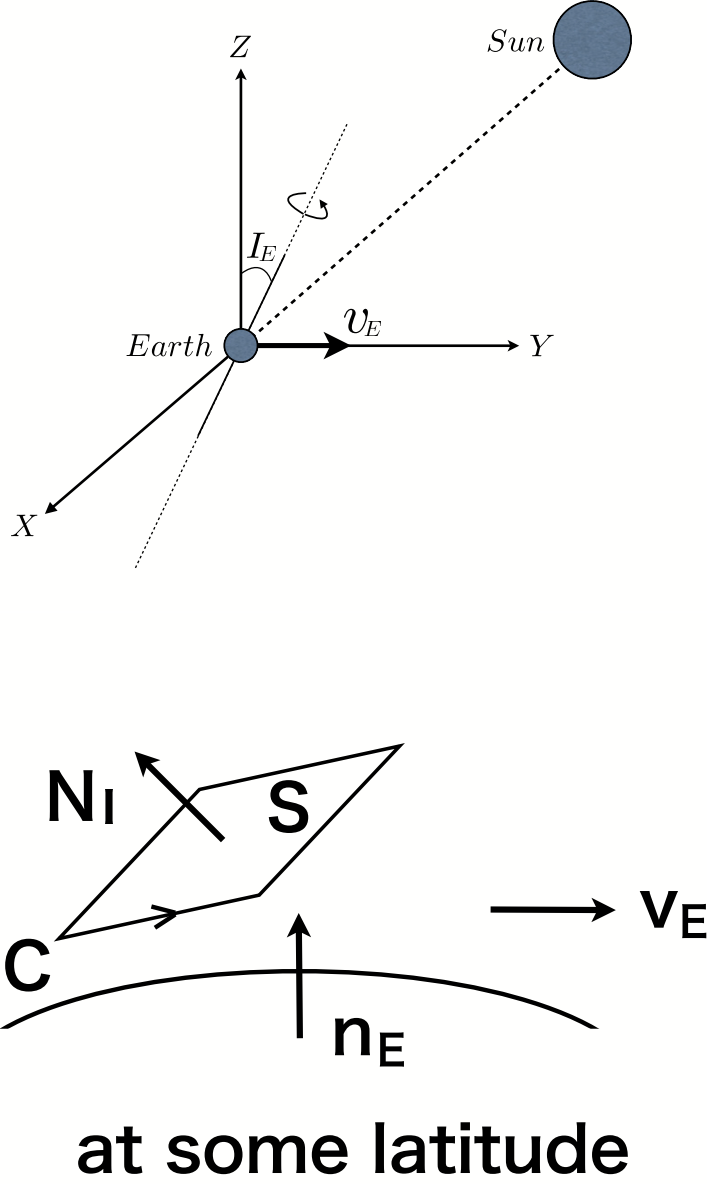


FIG. 1: Quantum interferometer on the Earth orbiting around the Sun. The orbital plane is chosen as the $X - Y$ plane. The Earth's axis and orbital velocity are denoted by I_E and v_E , respectively. Top panel: Earth orbiting around the Sun. Bottom panel: Interferometer at a certain time and place on the Earth. The latitude and the longitude are specified by \vec{n}_E , which rotates in an inertial frame around the Earth's axis and hence its direction changes also with the orbital motion of the Earth. The interferometer's direction \vec{N}_I also changes in an inertial frame as the Earth rotates.

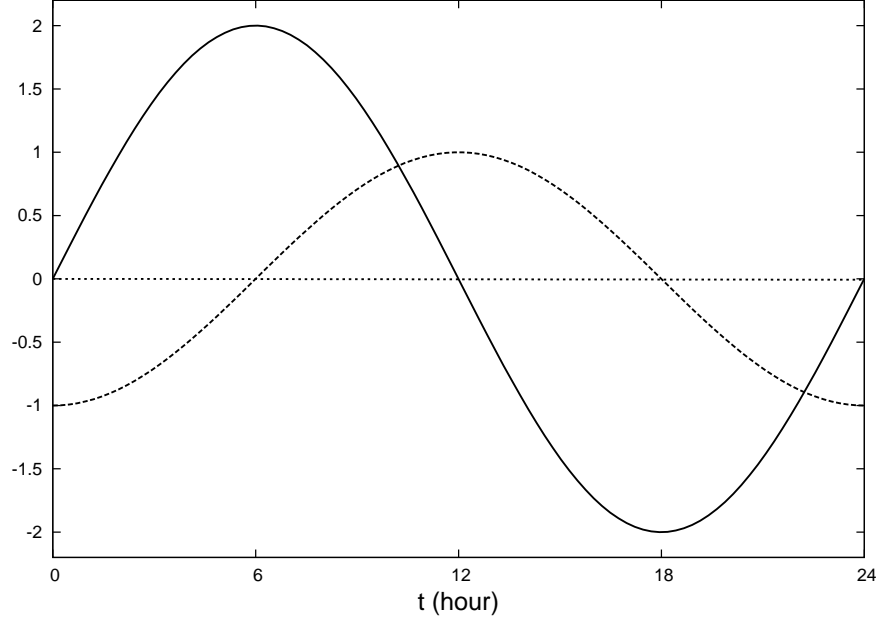


FIG. 2: Time variation in phase differences by CS effects on the vernal equinox day. The vertical axis (in arbitrary units) denotes $[3(\vec{v}_E \cdot \vec{n}_E)\vec{n}_E - \vec{v}_E] \cdot \vec{N}_I$ in Eq. (8). The quantum interferometer is located on the equator of the Earth. We consider three cases of the interferometer direction. The solid, dashed and dotted curves correspond to \vec{N}_I for a horizontal plane and two vertical ones (one facing the East and the other facing the North), respectively. The midnight is chosen as 0 hour. At midnight and at noon on the same day, CS effects on the phase difference vanish only for the horizontal case. This vanishing can be shown also by using Eq. (8), because $\vec{v}_E \perp \vec{n}_E \parallel \vec{N}_I$.

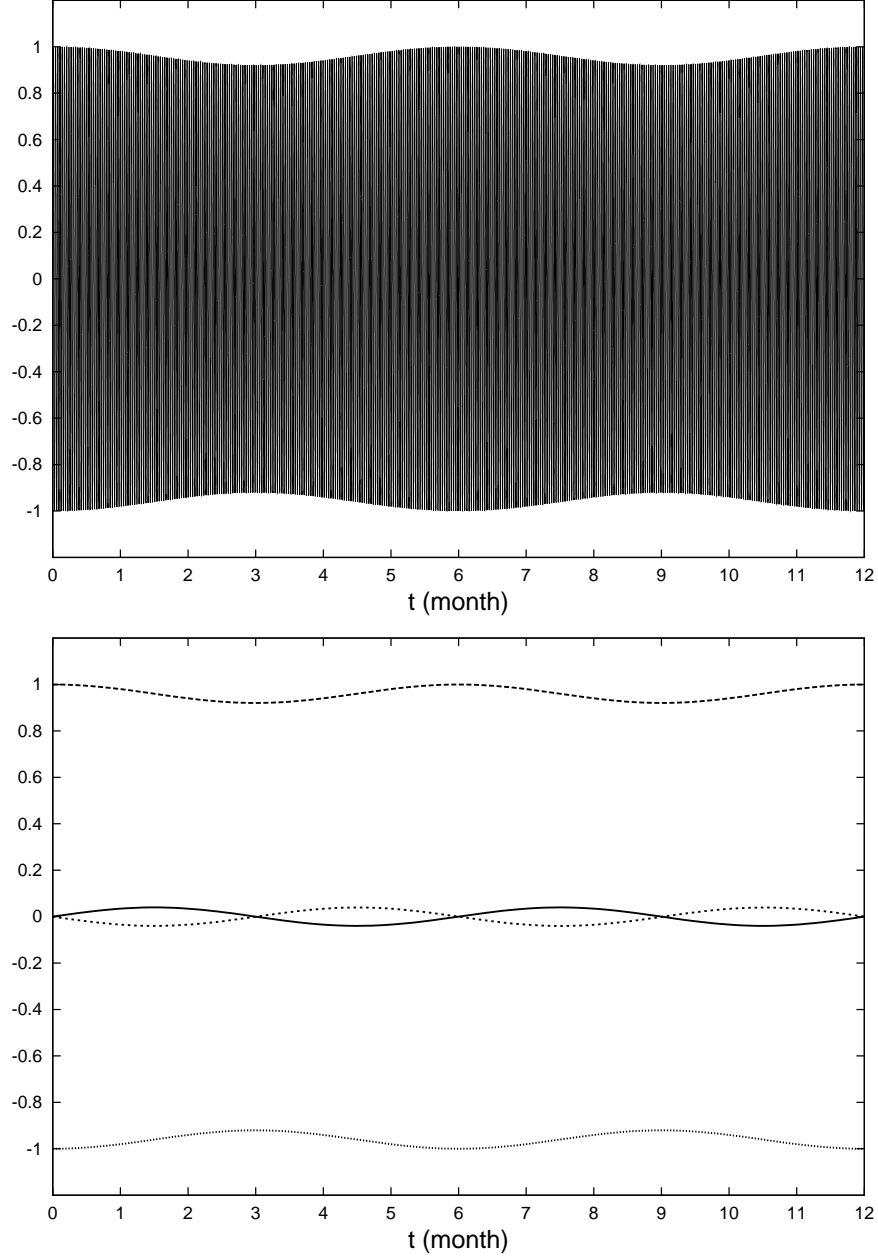


FIG. 3: Seasonal variation in phase differences by CS effects on the same quantum interferometer (on a horizontal plane) as the solid curve in Fig. 2. The winter solstice day is chosen as 0 month along the horizontal axis and the summer solstice corresponds to six months. Upper panel: full data points. Bottom panel: solid, long dashed, short dashed and dotted curves for 0, 6, 12 and 18 hours of each day, respectively.